North Carolina Junior Math Olympiad

NCMO Staff

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Welcome to the North Carolina Math Olympiad! You will have 3 hours to solve 5 problems. Each problem will be graded out of 7 points based on the completeness, clarity, and correctness of your solution.

The only tools permitted are writing utensils, erasers, and straightedge and compass if you have one (optional). For each solution page you wish to submit, write your ID, the problem number, and the page number for that solution in the top right corner:

Contestant ID 999
NCJMO Problem 5
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If you have no experience with proofs, simply write your answer as if you were explaining it to a friend: concisely, clearly, yet with no holes. Even if you don't think you have fully solved a problem, write down and submit whatever progress you have.

This will be a very challenging test, and you should be proud to solve even one problem. Try reading all of the problems and thinking about each one at least briefly, even if you decide to focus on one or two. If you ever you feel tired and need a break, feel free to leave your desk quietly and eat our provided snacks or stretch as needed (within the proctor's view). Good luck and have fun!

NCJMO 1. Cerena, Faith, Edna, and Veronica each have a cube. Aarnő knows that the side lengths of each of their cubes are distinct integers greater than 1, and he is trying to guess their exact values. Each girl fully paints the surface of her cube in Carolina blue before splitting the entire cube into $1 \times 1 \times 1$ cubes. Then,

- Cerena reveals how many of her $1 \times 1 \times 1$ cubes have exactly 0 blue faces.
- Faith reveals how many of her $1 \times 1 \times 1$ cubes have exactly 1 blue faces.
- Edna reveals how many of her $1 \times 1 \times 1$ cubes have exactly 2 blue faces.
- Veronica reveals how many of her $1 \times 1 \times 1$ cubes have exactly 3 blue faces.

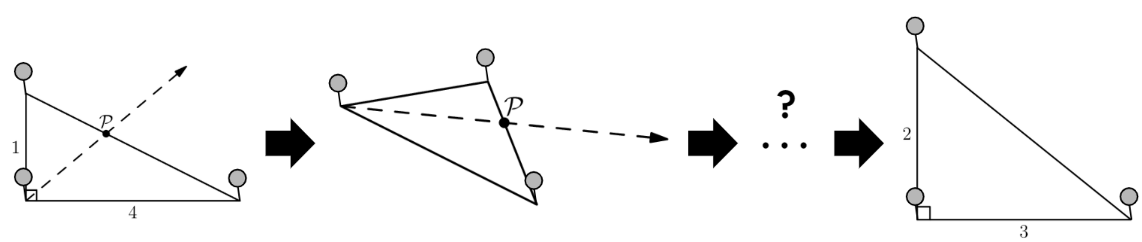
Whose side lengths can Aarnő deduce from these statements?

If your answer is (for example) "Cerena and Faith", you must (a) describe how Aarnő can deduce Cerena's and Faith's side lengths, and (b) explain why he *can't* deduce Edna's and Vernoica's side lengths.

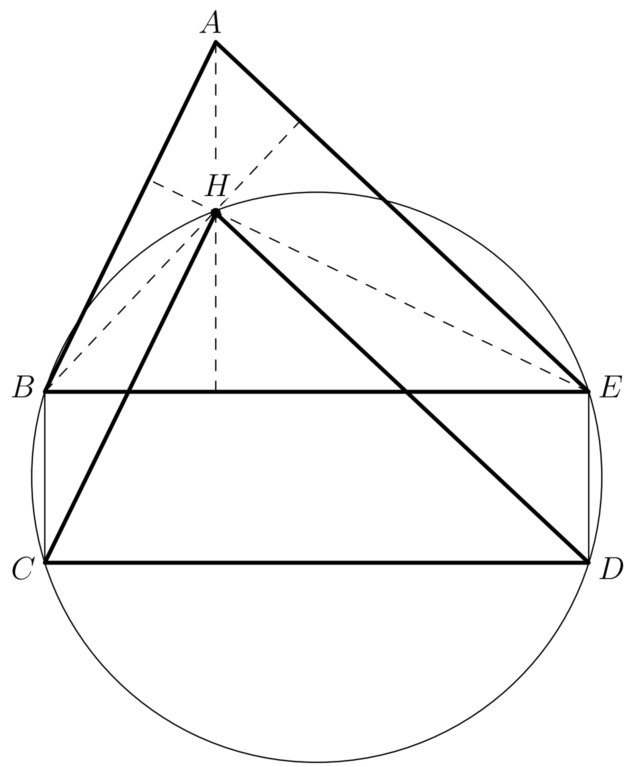
NCJMO 2. A collection of n positive integers, where repeats are allowed, adds to 500. They can be split into 20 groups each adding to 25, and can also be split into 25 groups each adding to 20. (A group is allowed to contain any amount of integers, even just one integer.) What is the least possible value of n?

If your answer is (for example) 2025, you must (a) describe a collection of 2025 positive integers that works, and (b) explain why the collection cannot be any smaller.

NCJMO 3. Alan has three pins that form a right triangle with legs 1 and 4 at first. Every move, he can pick any one of the pins, pick any new point $\mathcal{P}$ on the opposite side, and move the pin to its *reflection* across $\mathcal{P}$. After a series of moves, can the pins eventually form a right triangle with legs 2 and 3? Explain your answer.



NCJMO 4. In pentagon $ABCDE$, the altitudes of triangle ABE meet at point H. Suppose that $BCDE$ is a rectangle, and that B, C, D, E, and H lie on a single circle. Prove that triangles ABE and HCD are congruent.



NCJMO 5. Imagine that everyone in a room is wearing many colored wristbands. Each wristband has a single color, but there are many different colors of wristbands.

A "rainbow" is subset of the room where each person in the room shares a wristband color with at least one member of the rainbow. For example, the subset containing everyone is of course a rainbow, since everyone in the room shares a color with some member of the rainbow (namely, themself).

A "minimal rainbow" is a rainbow where removing any one of its members gives a non-rainbow. Prove that every person in the room is part of at least one minimal rainbow.