North Carolina Junior Math Olympiad

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Sponsors

NCJMO 1. Four players are playing a cooperative game. Each player is assigned a single secret integer they do not know, and receives the 3 integers assigned to the other players in a random order. Then, each player writes a single integer on a slip of paper.

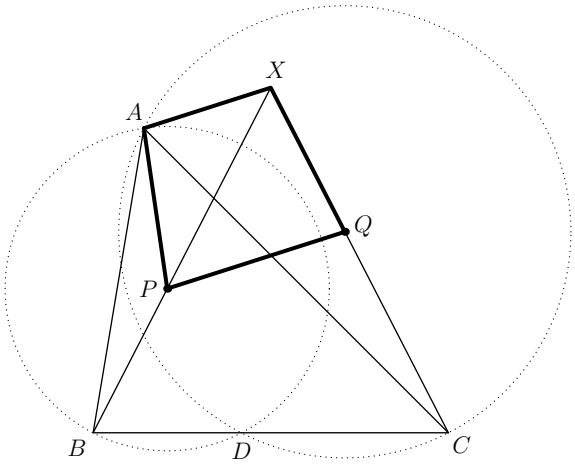
Finally, the players share their slips of paper, and must determine their own secret number. Show that the players are able to do so using an appropriate strategy, if they are only allowed to communicate before being assigned their secret integers, and through the 4 numbers on the slips of paper.

NCJMO 2. An equal number of boys and girls sit in a circle. Prove that the number of boys with a boy to his right equals the number of girls with a girl to her right.

NCJMO 3. An 8×8 chessboard is dissected along its gridlines into 2 connected pieces. Find, with proof, the greatest possible sum of their perimeters.

For full points, you must (a) draw or describe a dissection that attains your answer, and (b) prove that your answer is the greatest possible.

NCJMO 4. In triangle ABC, point D lies on side BC such that $\angle BAD = \angle CAD$. Let P be the center of the circle passing through A, B, and D; and let Q be the center of the circle passing through A, C, and D. If lines BP and CQ meet at X, prove that quadrilateral $APQX$ is an isosceles trapezoid.



NCJMO 5. Call a finite nonempty set *grizzy* if its elements have rational geometric mean. Let $\mathcal{S}$ be a grizzy set such that for any subset $\mathcal{T}$ of $\mathcal{S}$, exactly one of $\mathcal{T}$ and its complement in $\mathcal{S}$ is grizzy. Find all possible values for the number of elements in $\mathcal{S}$.

For full points, you must (a) describe sets $\mathcal{S}$ that attain all of your claimed values, and (b) prove that no other values are attainable.

Definitions.

- The *geometric mean* of $a_1, a_2, \ldots, a_n$ is defined as $\sqrt[n]{a_1 a_2 \ldots a_n}$; for example, the geometric mean of 8, 27, and 64 is $\sqrt[3]{8 \cdot 27 \cdot 64} = 24$.
- The *complement* of set $\mathcal{X}$ *in* set $\mathcal{Y}$ is the set of all elements in $\mathcal{Y}$ but not in $\mathcal{X}$; for example, the complement of $\mathcal{X} := \{1, 2\}$ in $\mathcal{Y} := \{1, 2, 3, 4, 5\}$ is $\{3, 4, 5\}$.