North Carolina Math Olympiad

NCMO Staff

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Sponsors

NCMO 1. An equal number of boys and girls sit in a circle. Prove that the number of boys with a boy to his right equals the number of girls with a girl to her right.

NCMO 2. An 8×8 chessboard is dissected along its gridlines into 2 connected pieces. Find, with proof, the greatest possible sum of their perimeters.

For full points, you must (a) draw or describe a dissection that attains your answer, and (b) prove that your answer is the greatest possible.

NCMO 3. Call a finite nonempty set *grizzy* if its elements have rational geometric mean. Let $\mathcal{S}$ be a grizzy set such that for any subset $\mathcal{T}$ of $\mathcal{S}$, exactly one of $\mathcal{T}$ and its complement in $\mathcal{S}$ is grizzy. Find all possible values for the number of elements in $\mathcal{S}$.

For full points, you must (a) describe sets $\mathcal{S}$ that attain all of your claimed values, and (b) prove that no other values are attainable.

NCMO 4. Determine, with proof, whether there exist real numbers $a, b > 1$ such that the quantity $\lfloor a^n \rfloor - \lfloor b^n \rfloor$ is a perfect power of 2 for infinitely many positive integers n.

NCMO 5. In $\triangle ABC$ with $AB = AC$ and incenter I, let E be the midpoint of side AC, and let F be the foot of the altitude from C to side AB. Prove that the circumcircles of $\triangle AEF$ and $\triangle BCI$ are tangent.

Definitions.

- The *geometric mean* of $a_1, a_2, \ldots, a_n$ is defined as $\sqrt[n]{a_1 a_2 \ldots a_n}$; for example, the geometric mean of 8, 27, and 64 is $\sqrt[3]{8 \cdot 27 \cdot 64} = 24$.
- The *complement* of set $\mathcal{X}$ in set $\mathcal{Y}$ is the set of all elements in $\mathcal{Y}$ but not in $\mathcal{X}$; for example, the complement of $\mathcal{X} := \{1, 2\}$ in $\mathcal{Y} := \{1, 2, 3, 4, 5\}$ is $\{3, 4, 5\}$.
- The *floor* $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x; for example, the floor of 3.14 is $\lfloor 3.14 \rfloor = 3$.
- A pair of geometric objects are *tangent* if they meet at exactly one point.
- The *incenter* of a triangle is the center of the unique circle internally tangent to all its sides; equivalently, it is the intersection of all its interior angle bisectors.