# NC(J)MO 2026 Results and Solutions

## Contents

# 1 Winners

## 1.1 NCMO In-State

| Name | Grade | School | P1 | P2 | P3 | P4 | P5 | Total |
|---|---|---|---|---|---|---|---|---|
| Watson Houck | 10 | Myers Park | 7 | 6 | 7 | 0 | 1 | 21 |
| Lingfei Tang | 11 | Cary Academy | 7 | 7 | 7 | 0 | 0 | 21 |
| Nelson Huang | 11 | NCSSM-Durham | 7 | 7 | 7 | 0 | - | 21 |
| Eric Zhu | 11 | Cary Academy | 7 | 6 | 7 | - | - | 20 |
| Srinivas Vijay Babu | 12 | NCSSM-Durham | 6 | 6 | 6 | - | - | 18 |

## 1.2 NCMO Out-of-State

| Name | P1 | P2 | P3 | P4 | P5 | Total |
|---|---|---|---|---|---|---|
| Alexander Wang | 7 | 6 | 1 | 7 | 7 | 28 |
| Isaac Chan-Osborn | 7 | 7 | 7 | 0 | 7 | 28 |
| Bruno Fiszelew | 7 | 7 | - | - | 7 | 21 |
| Shunyao Yan | 7 | 7 | 0 | - | 7 | 21 |
| Alex Liu | 7 | 5 | 0 | - | 7 | 19 |

## 1.3 NCJMO In-State

| Name | Grade | School | P1 | P2 | P3 | P4 | P5 | Total |
|---|---|---|---|---|---|---|---|---|
| Daniel Liu | 9 | Cary Academy | 7 | 7 | 7 | 7 | 1 | 29 |
| Andrew Chen | 7 | Smith Middle | 7 | 7 | 6 | 7 | 0 | 27 |
| Tarun Vedam | 10 | Palisades High | 7 | 7 | 5 | 7 | - | 26 |
| Ankush Dey | 9 | Cox Mill High | 7 | 7 | 7 | 1 | 2 | 24 |
| Bryan Zhong | 12 | NCSSM-Durham | 7 | 7 | 7 | 2 | 0 | 22 |

## 1.4 NCJMO Out-of-State

| Name | P1 | P2 | P3 | P4 | P5 | Total |
|---|---|---|---|---|---|---|
| Najeel So | 7 | 7 | 6 | 7 | 7 | 34 |
| Aaron Su | 7 | 7 | 3 | 7 | - | 24 |
| Zixuan Niu | 7 | 7 | 3 | 7 | - | 24 |
| Sophia Razafimahefa | 7 | 7 | 6 | - | 0 | 20 |
| Ray Zhao | 7 | 7 | 3 | - | - | 17 |

## 1.5 Special Awards

# 2 Solutions

## 2.1 NCJMO 1, Aaron Wang

**Problem.** Four players are playing a cooperative game. Each player is assigned a single secret integer they do not know, and receives the 3 integers assigned to the other players in a random order. Then, each player writes a single integer on a slip of paper.

Finally, the players share their slips of paper, and must determine their own secret number. Show that the players are able to do so using an appropriate strategy, if they are only allowed to communicate before being assigned their secret integers, and through the 4 numbers on the slips of paper.

**Solution 1 (Author).** Have each player write the sum of their 3 numbers. When they share slips, everyone can deduce that the total of all 4 secret numbers is $\frac{1}{3}$ the sum of numbers on the slips. Then, each player can deduce their secret number as the total minus the sum they wrote.

**Solution 2 (Adapted from Haoran Wang's Submission).** Have each player convert their 3 numbers to binary and smash them together into a single number, using the digit 2 to mark where each number ends, and the digit 3 for any negative signs. For example, a player given $-5$, $-2$, and 10 will write down

$$-101_2, -10_2, 1010_2 \to 3101231021010.$$

As such, the players essentially share all 3 numbers they receive with each other, and so each player can deduce their secret number as whichever of the shared numbers they didn't receive initially.

## 2.2 NCJMO 2 / NCMO 1, Benny Wang

**Problem.** An equal number of boys and girls sit in a circle. Prove that the number of boys with a boy to his right equals the number of girls with a girl to her right.

**Solution 1 (Author).** The number of boys with a girl to his right equals the number of girls with a boy to her left. Subtracting from the equal totals of boys and girls, we see the number of boys with a *boy* to his right equals the number of girls with a *girl* to her left, which in turn equals the number of girls with a girl to her *right*—as we wanted!

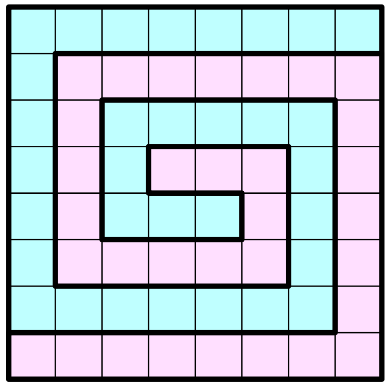
**Solution 2 (Alan Cheng).** Observe that swapping any pair of adjacent people preserves the difference in the numbers of boy-boy and girl-girl pairs. Hence, we can repeatedly swap until the boys and girls are alternating, at which point the statement clearly holds since there are no boy-boy or girl-girl pairs.

**Solution 3 (Jason Lee).** There are $n-1$ boy-boy pairs in a consecutive run of $n$ boys, and so the total number of boy-boy pairs is the total number of boys minus the number of consecutive runs of boys. Similarly, the number of girl-girl pairs is the total number of girls minus the number of runs of girls, and so it suffices to show there are the same number of boy runs and girl runs, which clearly holds true (it is true for any positive numbers of boys and girls sitting in a closed circle).

## 2.3 NCJMO 3 / NCMO 2, Jason Lee

**Problem.** An $8 \times 8$ chessboard is dissected along its gridlines into 2 connected pieces. Find, with proof, the greatest possible sum of their perimeters.

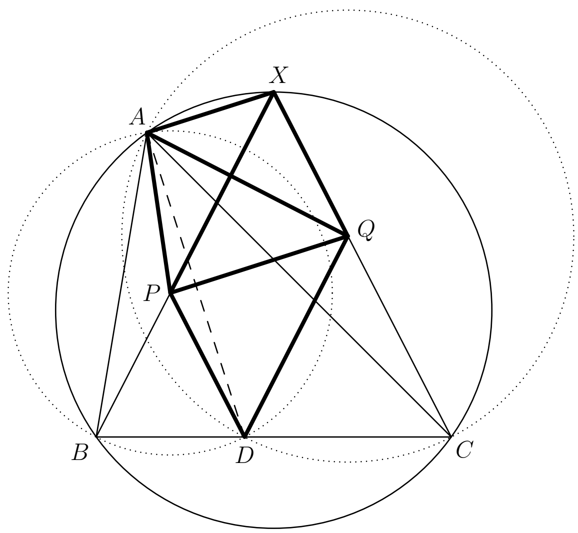
**Solution 1 (Author).** The answer is 132, constructed on the following page. Observe that a connected piece of $s$ squares has perimeter at most $2s+2$ by induction, since adding a single square adds at most 2 units of perimeter. Hence, if the board is split into $s_1$ and $s_2$ squares where $s_1 + s_2 = 64$, the sum of their perimeters is at most $(2s_1+2)+(2s_2+2) = 2(s_1+s_2)+4 = 2 \cdot 64 + 4 = 132$, as desired.



**Solution 2 (Contestants).** The sum in question is the board's outer perimeter of 32 plus 2 times the inner border $b$ between the pieces, and so it is equivalent to maximize $b$. Observe that $b$ can pass through each of the inner $7^2 = 49$ vertices at most once, and can only touch the outer perimeter at most 2 times, meaning $b \le 50$ and $32 + 2b \le 132$ as before.

## 2.4 NCJMO 4, Jason Lee

**Problem.** In triangle $ABC$, point $D$ lies on side $BC$ such that $\angle BAD = \angle CAD$. Let $P$ be the center of the circle passing through $A$, $B$, and $D$; and let $Q$ be the center of the circle passing through $A$, $C$, and $D$. If lines $BP$ and $CQ$ meet at $X$, prove that quadrilateral $APQX$ is an isosceles trapezoid.




**Solution 1 (Author and Benny Wang).** We wish to show that $\triangle APQ \cong \triangle XQP$. Observe that $\triangle APQ \cong \triangle DPQ$ by SSS, since $AP = DP$ and $AQ = DQ$ are common radii. Hence, it suffices to show that $\triangle DPQ \cong \triangle XQP$, or in other words that $DPXQ$ is a parallelogram.

But $\triangle BDP$ and $\triangle DCQ$ are isosceles with vertex angle $\angle BPD = 2\angle BAD = 2\angle DAC = \angle DQC$, and so $\triangle BDP \sim \triangle DCQ$. Hence, $\angle BDP = \angle DCQ \implies DP \parallel CX$ and $\angle DBP = \angle CDQ \implies BX \parallel DQ$, meaning $DPXQ$ is a parallelogram as desired.

**Solution 2 (Author and Royce Yao).** Salmon lemma implies $\triangle ABC \sim \triangle APQ$, and so $X$ is their spiral center and lies on both $(ABC)$ and $(APQ)$. In fact, $X$ is the major arc midpoint on $(ABC)$, as $\angle XBC = \angle PBD = \angle QCD = \angle XCB$ by Solution 1. Then $AD \perp AX$ as $AD$ is the internal bisector of $\angle BAC$; yet $AD \perp PQ$ as $\triangle APQ \cong \triangle DPQ$ again by Solution 1. So $AX \parallel PQ$ in $(APQX)$ as desired.

## 2.5 NCJMO 5 / NCMO 3, Grisham Paimagam

**Problem.** Call a finite nonempty set *grizzy* if its elements have rational geometric mean. Let $\mathcal{S}$ be a grizzy set such that for any subset $\mathcal{T}$ of $\mathcal{S}$, exactly one of $\mathcal{T}$ and its complement in $\mathcal{S}$ is grizzy. Find all possible values for the number of elements in $\mathcal{S}$.

**Solution.** The answer is all odds. Let $\Pi_{\mathcal{X}}$ denote the geometric mean of the elements in $\mathcal{X}$, such that a set $\mathcal{X}$ is grizzy if and only if $\Pi_{\mathcal{X}}$ is rational.

*Necessity.* If $|\mathcal{S}|$ is even, consider a subset $\mathcal{T}$ of size $\frac{1}{2}|\mathcal{S}|$ with complement $\mathcal{S} \setminus \mathcal{T}$ of the same size; the problem condition states that exactly one of $\Pi_{\mathcal{T}}$ and $\Pi_{\mathcal{S}\setminus\mathcal{T}}$ is rational. Now observe that $\Pi_{\mathcal{T}} \cdot \Pi_{\mathcal{S}\setminus\mathcal{T}} = \Pi_{\mathcal{S}}^2$; but $\Pi_{\mathcal{S}}$ is rational i.e. $\mathcal{S}$ is grizzy, since its complement the empty set cannot be grizzy by definition. Then $\Pi_{\mathcal{T}}$ and $\Pi_{\mathcal{S}\setminus\mathcal{T}}$ multiply to a rational number, meaning neither can be rational alone—contradiction.

*Sufficiency.* If $|\mathcal{S}|$ is odd, let $\mathcal{S} = \{2^{|\mathcal{S}|!!} < 3^{|\mathcal{S}|!!} < 5^{|\mathcal{S}|!!} < \cdots\}$, where $|\mathcal{S}|!! = |\mathcal{S}| \cdot (|\mathcal{S}| - 2) \cdot \ldots \cdot 1$ is divisible by each odd at most $|\mathcal{S}|$. Any subset of odd size is grizzy, since the product of its elements in any subset is a perfect $|\mathcal{S}|!!^{\text{th}}$ power; but no subset of even size is grizzy, since no subset of elements can multiply to a perfect square. Because $|\mathcal{S}|$ is odd, for any $\mathcal{T}$ the sizes $|\mathcal{T}|$ and $|\mathcal{S} \setminus \mathcal{T}|$ are even and odd in some order, meaning exactly one is grizzy as needed. So our $\mathcal{S}$ works.

## 2.6 NCMO 4, Grisham Paimagam

**Problem.** Determine whether there exist real numbers $a, b > 1$ such that the quantity $\lfloor a^n \rfloor - \lfloor b^n \rfloor$ is a perfect power of 2 for infinitely many positive integers $n$.

**Solution.** The answer is yes; in fact, there exists such $a$ for each $b$, say $b = 5.1$. The game plan will be to construct infinitely many nested intervals $\mathcal{I}_1 \supset \mathcal{I}_2 \supset \ldots$ with corresponding $n_1, n_2, \ldots$ such that for each index $k$, the quantity $\lfloor a^{n_k} \rfloor - \lfloor 5.1^{n_k} \rfloor$ is a power of 2 for all $a \in \mathcal{I}_k$.

- Set $n_1 = 1$, so that $\lfloor a^1 \rfloor - \lfloor 5.1^1 \rfloor = \lfloor a \rfloor - 5$ needs to be a power of 2. We might as well set that power to $2^0 = 1$, forcing $\lfloor a \rfloor = 6$ or $\mathcal{I}_1 = (6, 7)$.
- Then $\mathcal{I}_2^{n_2} \subset \mathcal{I}_1^{n_2} \subset (6^{n_2}, 7^{n_2})$. For large enough $n_2$, the interval $(6^{n_2}, 7^{n_2})$ contains $\lfloor 5.1^{n_2} \rfloor$ plus a power $2^c$, since $7^{n_2}$ grows exponentially faster than $6^{n_2}$. Hence, we can set

  $$\mathcal{I}_2^{n_2} = (\lfloor 5.1^{n_2} \rfloor + 2^c, \lfloor 5.1^{n_2} \rfloor + 2^c + 1)$$
  $$\text{i.e. } \mathcal{I}_2 = \left( \sqrt[n_2]{\lfloor 5.1^{n_2} \rfloor + 2^c},\ \sqrt[n_2]{\lfloor 5.1^{n_2} \rfloor + 2^c + 1} \right).$$

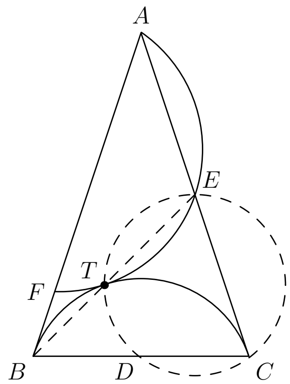
  (Technically, the upper $+1$ could exceed $7^{n_2}$ if we're unlucky; in that case we can re-pick $n_2$ and $c$ larger so that $(6^{n_2}, 7^{n_2})$ contains both $\lfloor 5.1^{n_2} \rfloor + 2^c$ and $\lfloor 5.1^{n_2} \rfloor + 2^c + 1$.)
- Generally, if $\mathcal{I}_{k+1} \subset \mathcal{I}_k^{n_k} = (x^{n_k}, y^{n_k})$, increase $n_{k+1}$ until $(x^{n_{k+1}}, y^{n_{k+1}})$ contains $\lfloor 5.1^{n_{k+1}} \rfloor$ plus a power of 2; we can do so no matter how close $x$ and $y$ are, since $y^{n_{k+1}}$ grows exponentially faster than $x^{n_{k+1}}$ as long as $x < y$. Then we can set $\mathcal{I}_{k+1}$ similar to how we did for $\mathcal{I}_2$ above.

Finally, since the $\mathcal{I}_k$'s are nested, we can intuitively pick an $a$ inside all of them, so that $\lfloor a^n \rfloor - \lfloor 5.1^n \rfloor$ is a power of 2 for the infinitely many $n = n_1, n_2, \ldots$, as desired. (Rigorously, one can show that the limit of the intervals' lower bounds is at most the limit of the upper bounds, a fact generalized in analysis as the *Nested Intervals Theorem*.)

## 2.7 NCMO 5, Alan Cheng

**Problem.** In $\triangle ABC$ with $AB = AC$ and incenter $I$, let $E$ be the midpoint of side $AC$, and $F$ the foot of the altitude from $C$ to side $AB$. Prove that the circumcircles of $\triangle AEF$ and $\triangle BCI$ are tangent.

**Solution 1 (Adapted from Bruno Fiszelew's Submission).**

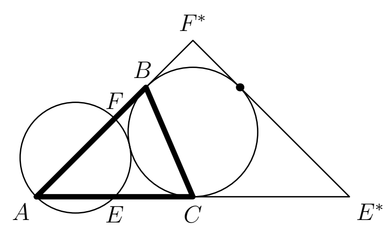



We show $T := BE \cap (CDE)$ is the tangency point, where $D$ is the midpoint of $BC$. First, $T \in (AEF)$ since $BT \cdot BE = BC \cdot BD = BA \cdot BF$ (as $ACDF$ is cyclic with center $E$). Next, $T \in (BCI)$ since

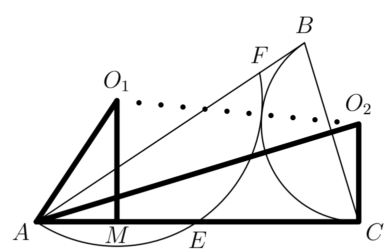
$$\measuredangle BTC = \measuredangle ETC = \measuredangle EDC = \measuredangle ABC = \measuredangle BCI + \measuredangle IBC = \measuredangle BIC.$$

Finally, to show $T$ is the tangency point, it suffices to show arcs $BT$ and $ET$ subtend the same angle: observe $(AEF)$ is tangent to $DE$, since $AE = EF$ means its circumcenter lies on the altitude from $E$ to $AF$, and so $\measuredangle BCT = \measuredangle DCT = \measuredangle DET = \measuredangle EAT$ as desired.

**Solution 2 (Benny Wang).** Inverting at $A$ with radius $AB$ preserves $(BCI)$ and maps $(AEF) \to E^*F^*$ where $\frac{A+E^*}{2} = C$ and $AF^* = E^*F^*$, implying tangency by symmetry across $CF^*$.




**Solution 3 (Jason Lee).** We use trigonometry to show the distance between the centers of $(AEF)$ and $(BCI)$ is the sum of their radii. Draw $AC$ horizontal, and let $\angle BAC := 2\theta$ and $(AEF)$ be unit.




Firstly, $(AEF)$ has radius $AO_1 = 1$ by assumption, and because $\angle AO_1E = 2\angle AFE = 2\angle FAE = 4\theta$, we have $MO_1 = \cos 2\theta = 1 - 2\sin^2\theta$ and $AM = EM = \sin 2\theta = 2\cos\theta\sin\theta$. Then $AC = 8\cos\theta\sin\theta$ and $CM = 6\cos\theta\sin\theta$, and since $\angle CAO_2 = \frac{1}{2}\angle BAC = \theta$, it follows that $CO_2 = AC\tan\theta = 8\sin^2\theta$.

Turning to the equation we want to show, observe that $O_1O_2$ is the hypotenuse of a right triangle with legs $CM$ and $|MO_1 - CO_2|$. Hence, by Pythagoras, it remains to show

$$(6\cos\theta\sin\theta)^2 + (1 - 10\sin^2\theta)^2 = (1 + 8\sin^2\theta)^2$$
$$1 + (6^2 - 2\cdot 10)\sin^2\theta + (10^2 - 6^2)\sin^4\theta = 1 + 2\cdot 8\sin^2\theta + 8^2\sin^4\theta$$

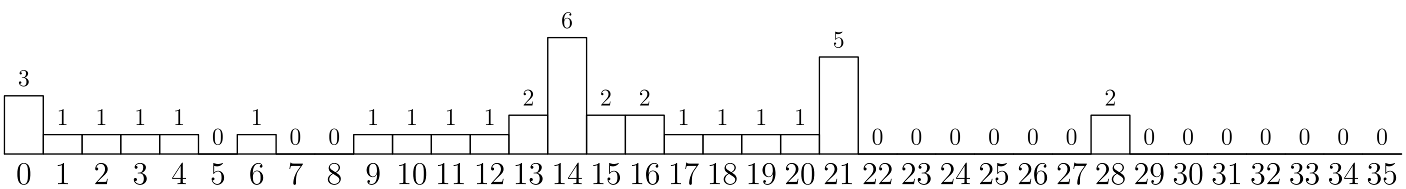
which, miraculously, holds true because $6^2 = 2(8+10)$ and $6^2 + 8^2 = 10^2$ (!).

# 3 Statistics

## 3.1 NCMO Solve Rates

| Pts | P1 | P2 | P3 | P4 | P5 |
|---|---|---|---|---|---|
| N/A | 2 | 3 | 11 | 24 | 17 |
| 0 | 2 | 1 | 9 | 10 | 10 |
| 1 | 3 | 3 | 2 | 0 | 3 |
| 2 | 0 | 3 | 3 | 0 | 0 |
| 3 | 1 | 2 | 2 | 0 | 0 |
| 4 | 0 | 1 | 0 | 0 | 0 |
| 5 | 0 | 4 | 0 | 0 | 0 |
| 6 | 2 | 7 | 1 | 0 | 0 |
| 7 | 25 | 11 | 7 | 1 | 5 |
| Avg | 5.5 | 4.5 | 2.0 | 0.2 | 1.1 |
| %5+ | 77 | 63 | 23 | 3 | 14 |

## 3.2 NCMO Score Histogram




## 3.3 NCJMO Solve Rates

| Pts | P1 | P2 | P3 | P4 | P5 |
|---|---|---|---|---|---|
| N/A | 4 | 4 | 3 | 20 | 25 |
| 0 | 10 | 6 | 9 | 12 | 16 |
| 1 | 3 | 6 | 11 | 5 | 3 |
| 2 | 1 | 0 | 10 | 2 | 1 |
| 3 | 0 | 3 | 3 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 2 | 0 | 0 |
| 6 | 5 | 1 | 4 | 0 | 0 |
| 7 | 23 | 26 | 4 | 7 | 1 |
| Avg | 4.3 | 4.4 | 2.2 | 1.3 | 0.3 |
| %5+ | 61 | 59 | 22 | 15 | 2 |

## 3.4 NCJMO Score Histogram